RPN 51

A MODEL OF ENERGETIC ION PRODUCTION BY INTENSE ELECTRON BEAMS*

PIIR-21-70

by

Sidney D. Putnam

July 1970

Physics International Company
2700 Merced Street
San Leandro, California 94577

*Submitted to Physical Review Letters

ABSTRACT

Recent experiments with intense linear electron beams have produced ions with energies exceeding the kinetic energy of the beam. A localized beam-pinch model explaining the phenomena is presented in this report.

Interest in linear-electron-beam, collective-field acceleration concepts has been renewed in the past year by the observations of Graybill and Uglum (Reference 1) and Rander, et al. (References 2 and 3). These investigators have obtained protons and deuterons up to 5 MeV in energy using 250-keV to 1-MeV electron beams with currents in the 50-kA range over a 50-cm-long drift chamber. Minimum average accelerating fields of 10^5 V/cm have thus been experimentally verified. The experiments have naturally generated speculation about the nature of the accelerating mechanism and the scaling of ion energies with beam and drift-chamber parameters. Wachtel and Eastlund (Reference 4) have suggested the Veksler "inverse Cerenkov" effect, while Rostoker (Reference 5) and, independently, Uglum, McNeill and Graybill (Reference 6) have proposed accelerated space-charge potential-well models. We propose a different mechanism, the localized-pinch model (Reference 7), whose predictions agree with presently established features of the experimental data.

In the experiments, an electron beam is injected through a thin metallic entrance (anode) window into a right-conducting cylindrical drift chamber with a small hole in the center at the downstream end. The beam and ions pass through the hole into a magnetic field where the beam and ions are separated; the ions are then diagnosed using time-of-flight, magnetic-spectroscopy, and nuclear-emulsion techniques. Various neutral gases at pressures from 10 to 300 microns are ionized by the beam. The salient features of the experimental data (References 1 to 3) are:

1. The peak ion energies are proportional to Z, the ion charge number, as would be the case if ions were accelerated by a stationary electrostatic field. The particle energy/Z is proportional to I^2, where I is the beam current. The experimental uncertainties allow a current dependence from $I^{3/2}$ to $I^{5/2}$.

2. The ion energy is nearly independent of filling-gas pressure over a factor-of-six variation in pressure.

3. The ion pulses are formed and accelerated after the fractional electrical neutralization,

$$f_e \equiv (-) \frac{\text{ion charge density}}{\text{electron charge density}} \geqslant \frac{1}{\gamma^2} = 1 - \beta_e^{\,2}$$

where γ is the electron energy/$m_o c^2$. The condition for radial-force neutralization and the onset of beam pinching is $f_e \sim 1/\gamma^2$ (Reference 8).

4. The proton-energy spread (FWHM) is less than 20 percent, the limit of the spectrometer resolution.

5. The total number of accelerated ions per ion pulse is in the range of 10^{12} to 10^{14} particles.

6. Multiple ion pulses have been reported by Rander, *et al.* (Reference 2).

In the localized-pinch model (LPM), a moving "slug" of ions always sees roughly the same accelerating field. By a slug of ions, we mean a localized enhancement in the ion density created near the anode window when the beam starts to pinch appreciably. With the experimental beam parameters, the pinching can occur so rapidly that nonadiabatic beam-envelope-collapse conditions are realized; i.e., the pinching occurs over distances of a few beam radii. It is the presence of the high-ion-density region that shorts out the radial electric (space-charge) field, allows pinching, and generates ion-accelerating fields. We are thus considering a self-synchronized accelerating process, which is illustrated in Figure 1.

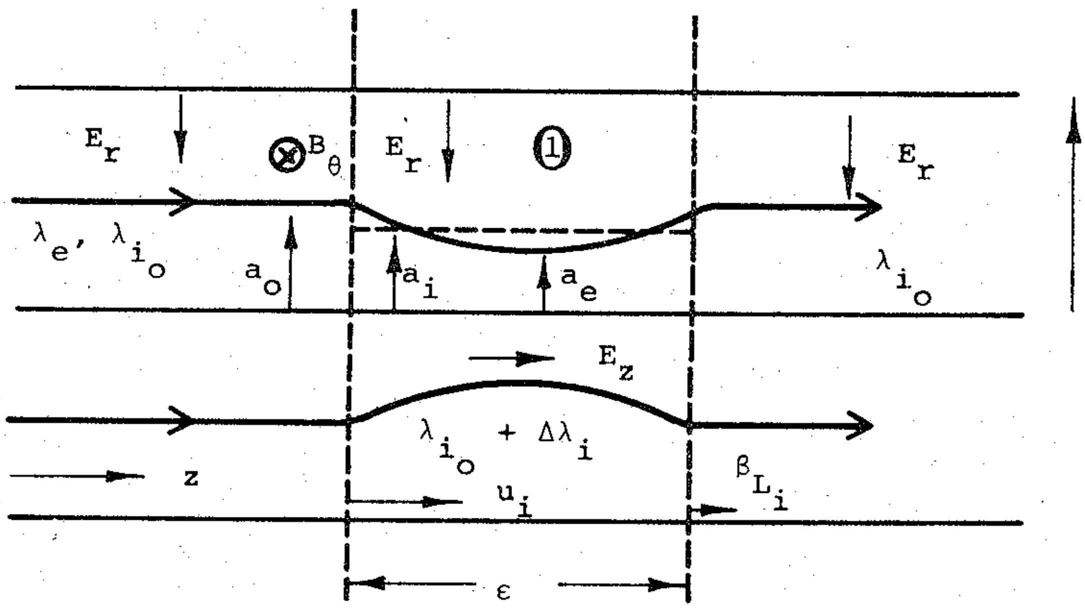

Figure 1 The ion-acceleration model

β_{L_i} = longitudinal β of ion slug = $v_{z_{ion}}/c$; ε = length of moving ion slug of region 1; λ_e = electron beam charge/length; λ_{io} = background ion charge/length; $\Delta\lambda_i$ = increment in ion charge/length in Region 1; a_o = ion and electron radius upstream from Region 1, a_e = beam envelope radius in Region 1; a_i = ion envelope radius in Region 1; B_θ = beam magnetic field; E_r = radial electric field; E_z = longitudinal (z-directed) electric field; R = radius of outer conducting pipe; $u_i = z - \beta_{L_i}\, ct$ = distance from upstream head of ion slug in the stationary frame of the ions.

In our idealized model we assume a zero rise length of the ion-density inhomogeneity, a constant beam current, $\beta_{L_i} << \beta_L$, and that β_{L_i} is approximately constant over times $\varepsilon/\beta_{L_i}c$. Moreover, we assume that the chamber end plates are "far away," that the background ion charge/length is constant, and that the beam and background ions are in equilibrium upstream from Region 1 at radius a_o. The ion and electron charge densities are taken uniform in radius out to the envelopes.

When the ion-envelope radius is constant $\approx a_o$ in Region 1, the beam-envelope equation can be written as

$$\frac{\partial^2 a_e}{\partial u_i^2} = \frac{1}{\beta_L^2 c^2}\left\{\left(\frac{-e}{m_o\gamma}\right)\left[E_r - \beta_L B_\theta\right]_{r=a_e} + \frac{C_e}{a_e^3}\right\}$$

$$\approx -\frac{2}{\gamma\beta_L^2}\left[\left(f_e^o + \frac{\Delta\lambda_i}{|\lambda_e|}\right)\frac{a_e}{a_o^2} - \frac{1}{\gamma_L^2 a_e}\right] + \frac{C_2}{\beta_L^2 c^2 a_e^3}, 0 \leqslant u_i < \varepsilon \qquad (1)$$

where

$\nu \equiv$ I (amperes)/(β_L) 17,000 (amperes)

$f_e^o = \lambda_{i_o} / |\lambda_e|$

$\gamma_L = (1 - \beta_L^2)^{-\frac{1}{2}}$

$\beta_L c$ = the average longitudinal velocity of the beam electrons

and

C_e = a constant proportional to the electron-beam emittance (Reference 9).

The E_z field along the beam axis (r = 0) resulting from envelope motion is

$$E_z \approx \frac{2\lambda_e}{a_e} \frac{\partial a_e}{\partial u_i}, \quad u_i > 0 \tag{2}$$

This field is always in the direction of electron flow during the beam-envelope contraction, in contrast to the case of an external field-driven-pinch collapse with ohmic current. Equation (1) can be reduced to quadrature, but for our purposes we merely estimate a turning length for the beam envelope, u_{i_t}, giving

$$E_z \text{ (V/cm)} \approx 4.4 \times 10^5 (\nu/\gamma)^{1/2} \frac{\nu}{a_o \beta_L} \left(f_e^{\,o} + \frac{\Delta\lambda_i}{|\lambda_e|} \right)^{1/2}, \quad 0 < u_i < u_{i_t} \tag{3}$$

Equation (3) underestimates the collapse velocity since we have assumed a constant ion-envelope radius. The maximum collapse velocity would obtain if $a_i \cong a_e$ in Region 1 (Reference 10).

Typical experimental parameters of $\nu/\gamma \approx 1$, $a_o \cong 1$ cm, $\gamma \cong 3$, $\beta_L \cong 0.91$, give $E_z \approx 7 \times 10^5$ V/cm if $f_e^{\,o} \cong 1/\gamma^2$ and $\Delta\lambda_i/|\lambda_e| \approx 1/\gamma^2$ (Reference 11). In this case, the electrons would lose all of their kinetic energy over a distance of the order of the beam radius. Our assumption that $\partial\beta_L/\partial u_i \approx 0$, $0 < u_i < u_{i_t}$ is violated. This example leads us to the concept of a "strong-inductance-dominated" pinch collapse. Generally speaking, if $\nu/\gamma << 1$, the pinch is slow and $u_{i_t} >> a_o$, whereas, if $\nu/\gamma \sim 1$, the pinch is strong-inductance dominated; i.e., the magnetically driven beam collapse is so fast that the "I dL/dt" longitudinal electric field (Reference 12) degrades the electron kinetic energy over distances of the order of the beam radius. Then λ_e increases with u_i and the current drops as electrons are lost radially, thus retarding further pinching (Reference 13). This condition is a "saturation" condition in that further increases in $\Delta\lambda_i$ do not appreciably increase E_z. The maximum E_z field value for a given current is E_z^{sat} (V/cm) $\approx$ 60 I(amperes)/a_o and is obtained by taking $\partial a_e/\partial t \approx c$ = velocity of light. In the example, $E_z^{sat} \cong 3 \times 10^6$ V/cm.

Our idealized model assumed the rise length of the ion-density inhomogeneity, ℓ_i, was zero, but, of course, any laboratory ion pulse would have a finite rise length. Nonadiabaticity requires that $\ell_i \lesssim a_o$ when $\nu/\gamma \sim 1$. We now develop a sufficient criterion for formation of a nonadiabatic collapse. Near the anode window, the longitudinal electrostatic field retards the beam front at injection into the neutral gas until collisional ionization generates approximate charge neutrality over a time scale, τ_N (Reference 14). When the beam front has passed the beginning of the "pinch-active" region at $z \approx R/2.4$, where the electrostatic field is now primarily radial, the beam starts to pinch as f_e exceeds $1/\gamma^2$. A nonadiabatic condition is generated if the pinching is fast enough so that

$$\frac{\partial j_{ion}}{\partial z} > \frac{\partial \rho_{ion}}{\partial t} \qquad (4)$$

where j_{ion} is the z-component of the ion current due to the E_z pinching field, and $\partial\rho_{ion}/\partial t$ is due to collisional ionization. Then pinching near the beginning of the pinch-active region is retarded, but immediately downstream it is enhanced because the ion background growth rate is increased; in other words, this process steepens the gradient of $\Delta\lambda_i$. If we estimate $\partial j_{ion}/\partial z$

using E_z from the beam radial contraction at constant current

$$\left(E_z \propto \frac{I}{a_e} \frac{da_e}{dt}\right)$$

over the time interval from $f_e = f_e^{o}$ to $f_e \approx 1$, Equation (4) gives

$$\nu \gtrsim 6 \left[\frac{\sqrt{\gamma - 1}}{\gamma^2 - 1} \; \frac{\gamma^4}{\beta_e} \; \frac{a_o(\mathrm{cm})}{\tau_N(\mathrm{nsec})} \left(\frac{m_i}{Zm_p}\right) \Big/ \ln\left(\frac{a_o}{a_1}\right)\right]^{2/3} \qquad (5)$$

where m_i/m_p is the ratio of the ion-to-proton mass and a_1 is the beam radius for $f_e \cong 1$. If the contraction is adiabatic, $\ln\,(a_o/a_1) \approx 1/2\,\ln\,[1 + C_i m_i/ZC_e\,\gamma m_o]$, with C_i the background-ion emittance. Equation (5) gives $\tau_N \gtrsim 40$ nsec for ion bunching in hydrogen using the maximum $\nu \approx 3$, $a_o = 1$ cm, $\ln\, a_o/a_i \approx 1$, in good agreement with the upper pressure cutoff of Graybill and Uglum, $\tau_N \approx 36$ nsec.

Growth of $\Delta\lambda_i$ will continue until $\beta_{L_i} >> \beta_{L_{io}}$, where $\beta_{L_{io}}$ refers to the velocity attained by background ions accelerated by E_z over a time $\varepsilon/\beta_{L_i}c$, or until the ion supply upstream is depleted, whichever occurs first. In our discussion, we have tacitly assumed that the background ions had zero net acceleration as the pulse passed. This is only true when the E_z field contribution from the variation of the ion charge/length (oppositely directed to the beam collapse field) is strong enough to completely decelerate the background ions slightly upstream from Region 1. While the ion pulse is growing near the anode, a net ion current, I_{ion}, flows behind the pulse proper, requiring an ion supply to maintain the current. When the supply is depleted an electrostatic well is re-established near the anode, which degrades the kinetic energy of the electrons and terminates further acceleration of the pulse. As collisional ionization continues near the anode, the whole process starts over again, i.e., multiple pulses are formed. If several charge species are present near the anode, the highest Z/m_i ratio ions (protons) are bunched and accelerated first.

The above arguments imply that the acceleration time, t_{acc}, is determined by the ion supply and effective pinching volume; i.e.,

$$\int_{t_1}^{t_{acc}+t_1} I_{ion}\, dt \sim \text{constant for given beam and drift-chamber parameters.}$$

The time, t_1, is the time at which a nonadiabatic collapse is achieved. This condition in turn implies that the acceleration length, L_{acc}, is independent of Z/m_i and that $t_{acc} \propto \sqrt{m_i/Z}$. Using $E_z \approx 7 \times 10^5$ V/cm from our example, the experimental ion energies of Reference 1 correspond to $L_{acc} \approx 7$ cm. The beam-front-velocity measurements of Rander et al. (Reference 3) also indicate that the acceleration occurs over a region a few centimeters long near the anode window.

The important questions of pulse growth times, stability, and lifetime require a more quantitative analysis; a linear-stability investigation of ion-density inhomogeneities in partially neutralized beams is in progress. Intuitively, we argue that radial stability at least requires that the outward magnetic force on the ions is not greater than the radially inward electric-field force, or

$$\beta_{L_i} < [1 - (f_e^o + \Delta\lambda_i/|\lambda_e|)]/\beta_L$$

If we take

$$\nu/\gamma \approx 1$$

$$\gamma \approx 3$$

$$\Delta\lambda_i/|\lambda_e| = f_e{}^o \approx 1/\gamma^2$$

the ions are radially stable for $\beta_{L_i} \lesssim 1$. We further note that in the strong-inductance-pinch condition,

$$E_z(r = 0)/E_z(r = a_o) \approx 1 + (2 \ell n R/a_o)^{-1}$$

which implies a small radial variation in E_z when $R/a_o >> 1$. The longitudinal synchronization or coherent acceleration length is limited by the loss of ions around $u_i = 0$ due to the finite rise length of E_z with u_i, which can be $<< a_o$ in the strong-inductance-pinch condition where E_z contributions from $\partial\lambda_e/\partial u_i$ are important.

In summary, the LPM predicts energetic-ion acceleration to occur after $f_e \cong 1/\gamma^2$, if the bunching criterion of Equation (5) is satisfied. For given beam parameters, this equation implies an upper limit on the pressure (shortest τ_N) and a lower pressure cutoff follows from the requirement that the time for attainment of $f_e \sim 1/\gamma^2$ must occur before the beam current starts to drop appreciably. The ion energy is $\approx Z\, E_z\, L_{acc}$, where E_z is given by Equation (3) for $eE_z \cdot a_o \leqslant$ beam kinetic energy, and L_{acc} is argued to be the same for all ion species. Since $E_z \propto \nu^{3/2}/\beta_L$ for fixed γ, the predicted current dependence falls within the range observed by Graybill and Uglum.

The author gratefully acknowledges the hospitality of the theoretical group of the Lawrence Radiation Laboratory, Berkeley, California, and many helpful conversations with Dr. A. M. Sessler and Professor Wulf Kunkel.

REFERENCES

1. S. Graybill and J. Uglum, J. Appl. Phys., Volume 41, 236 (1970).

2. J. Rander, B. Ecker, G. Yonas, and D. Drickey, Phys. Rev. Letters, Volume 24, 283 (1970).

3. J. Rander, B. Ecker, and G. Yonas, PIIR-9-70, Physics International Company, San Leandro, California, December 1969.

4. J. Wachtel and B. Eastlund, Bull. Am. Phys. Soc., II, Volume 14, 1047 (1969).

5. N. Rostoker, LPS 21, Laboratory for Plasma Studies, Cornell University, Ithaca, N.Y., July 1969; Bull. Am. Phys. Soc., II, Volume 14, 1047 (1969).

6. J. Uglum, W. McNeill and S. Graybill, Bull. Am. Phys. Soc., II, Volume 14, 1047 (1969).

7. S. Putnam, PIP-668, Physics International Company, San Leandro, California, September 1969; Bull. Am. Phys. Soc., II, Volume 14, 1048 (1969).

8. This feature is based on the collisional ionization rates used in Reference 1.

9. Emittance is a measure of the transverse beam energy. Its precise definition and the form of Equation (1) are given by I. Kapchinskij and V. Vladimirskij, Proc. Int. Conf. on High Energy Accelerators and Instrumentation, CERN, 1959, p. 274.

10. The quadrature of Equation (1) and estimates of u_{i_t} and the beam-collapse minimum radius are discussed in detail in S. Putnam, PIFR-105, Physics International Company, San Leandro, California, April 1970. See also J. Lawson, J. Elec. Cond., Volume 5, 146 (1958) for estimates of an approximate value for the radial collapse rate in the case $C_e = 0$, and $a_i = a_e$ during the collapse.

11. We have estimated β_L using an approximate steady-state relation:

$$\beta_L^2 \approx [\beta_e^2 + (\nu/\gamma)\ (1-f_e)]/(1 + \nu/\gamma).$$

REFERENCES (cont.)

12. L is the effective beam-chamber inductance/length.

13. Current-probe data of Reference 3 suggest a correlation between the time for attainment of $F_e \sim 1/\gamma^2$ and a dip in the net drift-chamber current.

14. See Reference 5 for discussion of this point.

15. Estimating τ_N near the anode is difficult because of possible electron-avalanche effects and the spread in primary-electron velocity.